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Quantum black holes as holograms in AdS braneworlds

Roberto Emparan[*]

Theory Division, CERN
CH-1211 Geneva 23, Switzerland
E-mail: roberto.emparan@cern.ch

Alessandro Fabbri

Dipartimento di Fisica dell'Università di Bologna & INFN sezione di Bologna
Via Irnerio 46, 40126 Bologna, Italy
E-mail: fabbria@bo.infn.it

Nemanja Kaloper

Department of Physics, Stanford University
Stanford, CA 94305-4060, USA
E-mail: kaloper@stanford.edu

Abstract: We propose a new approach for using the AdS/CFT correspondence to study quantum black hole physics. The black holes on a brane in an AdS_{D+1} braneworld that solve the classical bulk equations are interpreted as duals of *quantum-corrected* D-dimensional black holes, rather than classical ones, of a conformal field theory coupled to gravity. We check this explicitly in $D = 3$ and $D = 4$. In $D = 3$ we reinterpret the existing exact solutions on a flat membrane as states of the dual $2 + 1$ CFT. We show that states with a sufficiently large mass really are $2 + 1$ black holes where the quantum corrections dress the classical conical singularity with a horizon and censor it from the outside. On a negatively curved membrane, we reinterpret the classical bulk solutions as quantum-corrected BTZ black holes. In $D = 4$ we argue that the bulk solution for the brane black hole should include a radiation component in order to describe a quantum-corrected black hole in the $3 + 1$ dual. Hawking radiation of the conformal field is then dual to classical gravitational bremsstrahlung in the AdS_5 bulk.

Keywords: Extra Large Dimensions, AdS-CFT and dS-CFT Correspondence, Black Holes.

[*]Also at Departamento de Física Teórica, Universidad del País Vasco, Bilbao, Spain.

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Contents

1. Introduction

We propose here a connection between two seemingly unrelated problems in black hole theory: (i) the well-known problem of the backreaction from quantum effects on a black hole geometry, and (ii) the description of a black hole in an AdS braneworld, as in the Randall-Sundrum model with an infinite extra dimension, RS2 [1]. Quantum fields in a black hole background lead to particle production and black hole evaporation via Hawking radiation [2]. To leading order in perturbation theory, this yields an expectation value of the renormalized stress-energy tensor of quantum fields $\langle T_{\mu\nu}\rangle$, which includes quantum corrections. The backreaction of $\langle T_{\mu\nu}\rangle$ on the classical geometry modifies it according to the one-loop corrected Einstein's equation $G_{\mu\nu} = 8\pi G_4 \langle T_{\mu\nu}\rangle$. Unfortunately, the stress-energy tensor $\langle T_{\mu\nu}\rangle$ in a black hole spacetime can only be computed approximately, while determining its backreaction is even more difficult [3]. Only in dimensions $D < 4$ was it possible to find exact solutions [4]–[7].

On the other hand, an AdS braneworld consists of a bulk AdS_{D+1} space ending on a $D-1$-dimensional domain wall, or brane. A prototype is the RS2 model where AdS_5 ends on a 3-brane, which should model our 3+1 dimensional world. It is therefore natural to look for a suitable description of a black hole in this scenario. However, the attempts to find exact, static, asymptotically flat black hole solutions localized on the brane in $\mathrm{AdS}_{D+1>4}$, with regular horizons both on and off the brane, have come empty-handed to date (for published examples see, e.g., [8]–[12]). It has even been suggested that static, asymptotically flat, spherical black holes on the brane might not altogether exist in the RS2 model [9].[1] Contrasting this, exact static solutions localized on a 2-brane in AdS_4 have been found in [14, 15].

[1] Ref. [13] obtains a numerical solution for a static star on an RS2 brane.

Here we adopt the point of view that the difficulties in constructing these solutions are no mere accident, but are intricately related to the effects induced by quantum corrections. We use a modification of AdS/CFT correspondence [16] for the RS2 model [17]-[23] to connect both problems. Our main result is the following conjecture:

> *The black hole solutions localized on the brane in the AdS_{D+1} braneworld which are found by solving the classical bulk equations in AdS_{D+1} with the brane boundary conditions, correspond to quantum-corrected black holes in D dimensions, rather than classical ones.*

This conjecture follows naturally from the AdS/CFT correspondence adapted to AdS braneworlds. According to it, the *classical* dynamics in the AdS_{D+1} bulk encodes the *quantum* dynamics of the dual D-dimensional conformal field theory (CFT), in the planar limit of a large N expansion. Cutting the bulk with a brane introduces a normalizable D-dimensional graviton mode [1, 24], while on the dual side this same D-dimensional gravity mode is merely added to the CFT, which is also cutoff in the ultraviolet. Then, solving the classical $D+1$-dimensional equations in the bulk is equivalent to solving the D-dimensional Einstein equations $G_{\mu\nu} = 8\pi G_D \langle T_{\mu\nu}\rangle_{CFT}$, where the CFT stress-energy tensor incorporates the quantum effects of all planar diagrams. These include particle production in the presence of a black hole, and possibly other vacuum polarization effects.

This conjecture has implications in two directions. On the one hand, it allows us to view the brane-induced modifications of the metric of a D-dimensional black hole as quantum corrections from a CFT, a dual view that sheds light on both problems. On the other hand, we can use the conjecture to infer, from the known properties of the classical bulk solutions, the properties of the cutoff CFT coupled to gravity. Even if some of the conclusions are derived using the AdS/CFT correspondence, they are typically independent of the existence of a bulk dual: any strongly coupled CFT with a large number of degrees of freedom is likely to behave, when coupled to weak gravity, in a similar manner.

We submit the conjecture to the test by reinterpreting the exact solutions on the 2-brane in an AdS_4 braneworld [14, 15] as quantum-corrected, gravitating CFT states in the dual $2+1$ theory, either with or without a negative cosmological constant in $2+1$ dimensions, Λ_3. As is typical in tests of the AdS/CFT correspondence, the calculations on the CFT side can only be performed at weak 't Hooft coupling, often at the one-loop order only, and therefore comparisons with the strongly coupled dual of the classical bulk theory, which includes all planar diagrams, are difficult. Even then, we find some instances where the equivalence between the results at weak and strong coupling holds to a great degree of detail.

An interesting spin-off of the analysis is a realization of *quantum censorship of conical singularities*, which we argue is a generic effect independent of the AdS/CFT duality. Gravity in $2+1$ dimensions is known to describe massive particles in terms of conical singularities [25]. We find that when quantum corrections from a CFT are included, the singularity of a sufficiently massive particle is dressed by a regular horizon. This result is in fact true independently of whether the CFT is strongly or weakly coupled, and acts more efficiently when it has a large number of degrees of freedom.

Since we have a detailed description of the solutions in the AdS_4 braneworld, we can apply it to describe the objects which arise in the cutoff CFT. When $\Lambda_3 = 0$, the theory is characterized by three mass scales: the UV cutoff of the CFT, μ_{UV}, the 4D Planck mass and the 3D Planck mass, in ascending order. These scales naturally organize the range of CFT configurations into three categories: (i) the familiar light CFT states, with masses below the CFT cutoff, which *are not* black holes because of the quantum uncertainty-induced smearing; (ii) states with masses between the CFT cutoff and the 4D Planck mass, which also *are not* black holes because of quantum smearing and may receive large quantum corrections in the bulk; and (iii) black holes, which are the states with masses above the 4D Planck mass. These black holes may be smaller than the CFT length cutoff, $\hbar/\mu_{UV}$, but their description should be reliable since both the bulk and the $2+1$ gravity corrections are small. Our argument that the cutoff CFT can be trusted to distances much shorter than the UV cutoff is analogous to a familiar situation in string theory [26], suggesting that the intermediate mass states and light black holes behave as CFT solitons.

A negative cosmological constant $\Lambda_3 < 0$, allows for classical BTZ black holes [27]. Although the AdS/CFT duality is not fully understood for the case of negatively curved branes, we find that the solutions localized on the 2-brane are naturally interpreted as BTZ black holes with CFT quantum corrections, which are in equilibrium with a thermal bath in AdS_3. There are other localized solutions, all with mass less than $M_{max} = 1/(24G_3)$, with different features, but we find explanations for all of them within the context of our conjecture. Black holes of mass larger than M_{max} are delocalized black strings occupying an infinite region of the bulk, and it is unclear how to describe them within the confines of the $2+1$ theory; in fact, it is likely that such a description should not be possible in terms of only local physics.

In the physically more relevant case of a 3-brane in AdS_5 we can not go into a similar level of detail since there are no exact solutions, and classical gravity in $3+1$ dimensions is dynamical. However we can still explore the consequences of our conjecture in a semi-quantitative manner. The description in terms of a CFT coupled to gravity is not reliable until the horizon is larger than the ultraviolet cutoff of the CFT, i.e., the black hole is sufficiently heavy. For these black holes, the CFT+gravity theory allows us to reinterpret the alleged obstruction for finding a static black hole [9] as a manifestation of the backreaction from Hawking effects. The analysis of the trace anomaly of the CFT stress tensor allows us to make this point precise. As long as the anomaly is consistent with the asymptotic AdS_5 geometry, the conformal symmetry of the dual CFT is valid in the infrared, and so there is no mass gap. Hence any black hole at a finite temperature will emit CFT modes as a thermal spectrum of Hawking radiation, which on the bulk side is captured by a deformation of the bulk geometry close to the brane, caused by the black hole sourcing the classical gravity equations. We illustrate this to the leading order on the CFT side by showing that the backreaction from Hawking radiation, encoded in the form of a Vaidya-type far-field solution, is consistent with the CFT anomaly. We also discuss the dual bulk picture of Hawking radiation that arises from our conjecture. Within this interpretation, the difficulties encountered in the ongoing quest for the black hole localized on the 3-brane in AdS_5 are viewed as a natural, subleading quantum correction to the classical solution, rather than as a no-go theorem for the existence of classical braneworld black holes.

2. AdS/CFT duality for AdS braneworlds

We begin with a brief review of several aspects of the two dual descriptions that are relevant for our conjecture [16]-[23]. Since we want to discriminate between classical and quantum effects, we retain $\hbar$ in our formulas, while setting $c = 1$. Then, the D-dimensional Newton's constant G_D, Planck length ℓ_D, and Planck mass M_D are related to each other as

$$G_D = \frac{\ell_D^{D-3}}{M_D}, \qquad M_D = \frac{\hbar}{\ell_D}. \tag{2.1}$$

In AdS braneworlds the $D+1$ dimensional bulk Newton's constant and the bulk cosmological constant $\Lambda_D = -D(D-1)/2L^2$ together determine the Newton's constant induced on the D-dimensional brane as

$$G_D = \frac{D-2}{2L} G_{D+1}. \tag{2.2}$$

The precise details of the dual CFT depend on the specifics of the string/M-theory construction that yield the AdS background. Here we only need to know the effective number of degrees of freedom of the CFT, g_*. For $D = 4$, the dual pair are IIB string theory on $AdS_5 \times S^5$ of radius $L \sim \ell_{10}(g_s N)^{1/4}$ and $\mathcal{N} = 4$ SU(N) super Yang-Mills theory, while for $D = 3$, the dual pair are M-theory on $AdS_4 \times S^7$ and the (poorly known) theory describing the worldvolume dynamics of a large number N of M2 branes. In these cases

$$\begin{aligned} g_* &\sim N^2 \sim \left(\frac{L}{\ell_5}\right)^3 \sim \left(\frac{L}{\ell_4}\right)^2 \qquad (D=4)\,, \\ g_* &\sim N^{3/2} \sim \left(\frac{L}{\ell_4}\right)^2 \sim \frac{L}{\ell_3} \qquad (D=3)\,, \end{aligned} \tag{2.3}$$

where we have used (2.2) to get the final expressions. g_* is taken to be a large number, in order to keep small the quantum corrections to the supergravity approximation to string/M-theory. For the CFT, this is a large N limit where planar diagrams give the leading contribution.

The introduction of the brane that cuts off the AdS bulk implies that very high energy states of the dual CFT are integrated out, and the conformal invariance of the theory is broken in the ultraviolet. However, the breaking washes into the low energy theory only through irrelevant operators, generated by integrating out the heavy CFT states at the scale $\mu_{UV} \sim \hbar/L$. In the infrared, at energies $E < \mu_{UV}$, the effects of the conformal symmetry breaking are suppressed by powers of E/μ_{UV}. Cutting off the bulk yields also a normalizable graviton zero mode localized on the brane; this same D-dimensional gravity mode is added to the dual theory. However, note that the CFT cutoff μ_{UV} is *not* equal to the induced D-dimensional Planck mass. Instead,

$$\mu_{UV} \sim \frac{M_4}{\sqrt{g_*}} \quad (D=4)\,, \qquad \mu_{UV} \sim \frac{M_3}{g_*} \quad (D=3)\,, \tag{2.4}$$

which is much smaller than the Planck mass on the brane. The formulae above can be written for any AdS space and can be viewed as a definition of a cutoff CFT, although they do not guarantee the existence of its UV completion. We will use them bearing this in mind.

3. Quantum black holes on flat branes in $2+1$ dimensions

For the case of $D=3$, the exact four-dimensional solutions constructed in [14] yield the following metric on the 2-brane,

$$ds^2_{\text{brane}} = -\left(1-\frac{r_0}{r}\right)dt^2 + \left(1-\frac{r_0}{r}\right)^{-1}dr^2 + r^2 d\varphi^2 \,. \tag{3.1}$$

The parameter r_0 fixes the position of the horizon, and is determined by the mass M. In a locally asymptotically flat space in $2+1$ the mass is given by the conical deficit angle at infinity, $\delta_\infty = 8\pi\, G_3 M = 8\pi\, M/M_3$. It was shown in [14] that such a deficit angle is indeed present in (3.1), leading to[2]

$$M = \frac{M_3}{4}\left(1-\frac{\sqrt{1+x}}{1+\frac{3}{2}x}\right), \tag{3.2}$$

where x is defined by

$$x^2(1+x) = \frac{r_0^2}{L^2}\,. \tag{3.3}$$

These expressions define the horizon size r_0 as a function of the mass M in parametric form. The mass varies from $M=0$ ($r_0=0$) up to a maximum,

$$M_{\max} = \frac{1}{4G_3} = \frac{M_3}{4}\,, \tag{3.4}$$

which comes from the constraint that the deficit angle δ_∞ be smaller than 2π. For small masses $M \ll M_3$

$$r_0 \simeq \frac{4M}{M_3}\,L \ll L\,, \tag{3.5}$$

while for the masses near $M_{\max}$

$$r_0 \simeq \frac{8L}{27\,(1-M/M_{\max})^3} \gg L\,. \tag{3.6}$$

The presence of the horizon at $r=r_0$ may appear as a surprise since it is known that there are no asymptotically flat vacuum black holes in 2+1 dimensions [25]. But (3.1) is not a vacuum solution. Following our conjecture, it must admit an interpretation as a quantum-corrected solution of the $2+1$ CFT+gravity system. To see this, note that the general relation between the horizon radius and the mass is of the form $r_0 = L\,f(G_3 M)$, with

[2] In the notation of [14], M_3 was the mass as measured on the brane, and M_4 the mass measured in the bulk. They were shown to be the same, $M_3 = M_4$. Here we denote them by M, reserving M_3 and M_4 for the three- and four-dimensional Planck masses, as in eq. (2.1).

$f(G_3M)$ obtained from (3.2) and (3.3). In order to correctly identify quantum-mechanical effects we express the results in terms of only those variables which are meaningful in the dual CFT+gravity description. Using (2.1), (2.2) and (2.3) we can write $L \sim \hbar g_* G_3$, so

$$r_0 \sim \hbar g_* G_3\, f(G_3M)\,. \tag{3.7}$$

The appearance of $\hbar$ is a clear fingerprint of the quantum origin of the horizon viewed from the $2+1$ perspective. This is in complete agreement with our conjecture: since there are no horizons in the classical $2+1$ theory, any that are found must be purely quantum-mechanical in origin. The classical theory does not contain any length scale (G_3M is dimensionless), and only with the introduction of $\hbar$ can we form one, namely the Planck length $\ell_3 = \hbar G_3$, which sets the scale for r_0.

We can test the conjecture in more detail. The solution (3.1) can be formally obtained in the dual $2+1$ CFT coupled to gravity from the quantum-mechanical backreaction on the spacetime of a particle of mass M. Beginning with the conical geometry corresponding to a localized CFT lump representing a point particle, with deficit angle $\delta_\infty = 8\pi M/M_3$, one can compute the Casimir stress-energy and find its backreaction on the metric. Such a solution was indeed discovered almost a decade ago in [28] for the case of a weakly coupled scalar CFT. Its Casimir stress-energy was computed in [29] as

$$\langle T^\mu{}_\nu\rangle = \frac{\hbar\alpha(M)}{r^3}\,\mathrm{diag}(1,1,-2)\,, \tag{3.8}$$

where

$$\alpha(M) = \frac{1}{128\pi}\int_0^\infty \frac{du}{\sinh u}\left(\frac{\cosh u}{\sinh^3 u} - \frac{1}{(1-4G_3M)^3}\frac{\cosh[u/(1-4G_3M)]}{\sinh^3[u/(1-4G_3M)]}\right). \tag{3.9}$$

Using this stress-energy tensor to calculate the backreaction on the conical spacetime, ref. [28] found the metric (3.1), with $r_0 = 4\pi\hbar\,\alpha(M)/M_3$. In our case the CFT has a large number of degrees of freedom g_*, each of whom contributes to the Casimir stress-energy tensor. Thus we expect to find $r_0 = \mathcal{O}(1)\,\hbar g_*\alpha(M)/M_3$ where the $\mathcal{O}(1)$ factors can only be calculated when the exact description of the strongly-coupled CFT is known. Moreover, we can not expect the mass dependence of this r_0 to agree precisely with that of (3.3) — among other things, we have not even included the contribution from fermions to $\langle T^\mu{}_\nu\rangle$. Nevertheless, we may hope for some simplification in the limiting cases $M \ll M_3$ and $M \to M_3/4$. In the former limit,

$$\alpha(M) = \mathcal{O}(1)\frac{M}{M_3}\,, \tag{3.10}$$

so

$$r_0 = \mathcal{O}(1)\hbar g_*\frac{M}{M_3^2} = \mathcal{O}(1)\frac{M}{M_3}L\,, \tag{3.11}$$

which exactly reproduces eq. (3.5) up to $\mathcal{O}(1)$ coefficients. In the limit $M \to M_3/4$, the integrand in (3.9) is strongly peaked at $u = 0$ and $\alpha(M)$ can be computed using the saddle-point method,

$$\alpha(M) = \frac{\mathcal{O}(1)}{(1-4G_3M)^3}\,, \tag{3.12}$$

so the backreaction from the CFT results in

$$r_0 = \mathcal{O}(1) \frac{\hbar g_*}{M_3(1-4G_3M)^3} = \mathcal{O}(1) \frac{L}{(1-M/M_{\max})^3}\,, \tag{3.13}$$

which again reproduces the precise parametric dependence in eq. (3.6).

Alternatively, one can compare (3.8) with the stress-energy tensor computed directly from the metric (3.1),

$$T^\mu{}_\nu = \frac{1}{16\pi G_3} \frac{r_0}{r^3} \,\mathrm{diag}(1,1,-2)\,. \tag{3.14}$$

Both (3.8) and (3.14) have the same structure and radius dependence, so they determine the same geometry. The equivalence is completed by noting that, taking g_* times (3.8), and comparing to (3.14), we find $\hbar g_* \alpha \sim r_0/G_3$, as expected. This formally confirms the equivalence between the classical construction in AdS_4 and the quantum-corrected $2+1$ solution. The quantum corrections are completely due to Casimir-like vacuum polarization, rather than backreaction from Hawking radiation, since the classical solutions are not black holes to begin with. The Casimir effect acts here as a quantum censor, hiding the classical conical singularity behind a horizon.

The agreement between the calculations in the two sides of the conjecture is striking, given their completely different nature (classical vs. quantum), and we believe that it provides a strong argument in favor of the AdS/CFT correspondence in the context of AdS braneworlds, beyond the linearized calculation of [21]. One may ask whether the agreement is just a consequence of some common symmetry underlying both problems. This does not seem to be the case. Conformal invariance is present on both sides: since the bulk AdS is empty, it influences the brane only through the conformal Weyl tensor. However, conformal symmetry alone only determines the radial dependence r^{-3} of the stress tensor (recall that the classical $2+1$ theory has no length scale), and its traceless character. Neither the particular structure $\mathrm{diag}(1,1,-2)$, nor the dependence on the dimensionless quantity M/M_3, are fixed by conformal invariance.

So far we have been focusing on the mathematical side of our conjecture and ignoring the interpretation of the solutions (3.1). However, since we have argued that the solutions (3.1) are quantum-mechanical in origin, we must ask to what extent the description of a state of mass M based on (3.1) is physically valid. In particular, in the limit of small masses the curvature of the solution will be very large outside of the horizon, indicating that higher-order curvature corrections will invalidate the solution (3.1) already in a region larger than the horizon size.

To understand the physics of the solutions (3.1), note that the states of the CFT+gravity theory are defined by *three* scales: the CFT cutoff $\mu_{UV} \sim \hbar/L$ on the low end, the 3D Planck mass M_3 on the high end, and the 4D Planck mass M_4 in between. While M_4 is an obvious scale from the bulk side, from the viewpoint of the dual CFT coupled to $2+1$ gravity its presence is slightly mysterious. There, M_4 emerges because of the large number of CFT degrees of freedom, as $M_4 \sim M_3/\sqrt{g_*}$. Its importance can be seen as follows. Any solution of a given mass M is characterized by two length scales: the horizon radius r_0 and the Compton wavelength $\lambda_C = \hbar/M$. If $\lambda_C > r_0$, the solution cannot be a black hole, because quantum effects smear it over a volume larger than the horizon, but if $r_0 > \lambda_C$, the solution is a black hole, since quantum-mechanical fuzzying up is not sufficient to conceal the horizon. On the bulk side, this simply means that the description of this object by a *classical* metric in AdS space is not appropriate, and that one should instead use wave packets delocalized over λ_C as in quantum mechanics. Viewed from the bulk it is clear that the mass scale for the crossover is M_4. Translated into the $2+1$ description, this is the same value at which $r_0 \sim \lambda_C$: when $M \sim M_4 \ll M_3$, (3.5) and (2.2) imply $r_0 \sim LM_4/M_3 \sim \hbar/M_4 \sim \lambda_C$. Thus, M_4 is consistently the threshold scale for black hole formation. Above this scale, the curvature near the horizon is sub-Planckian, and the semiclassical geometry (3.1) becomes reliable all the way down to the black hole horizon r_0.

Since for $M > M_4$ the leading CFT corrections are large enough to give rise to a horizon, one may worry that higher order corrections may be very large as well, and render the leading approximation meaningless. Again, this does not occur. The higher-order effects in the $2+1$ description correspond to one-loop quantum effects (Hawking radiation) in the bulk. The black hole temperature is $T \sim \hbar/r_0$, and when $M > M_4$, $\hbar/r_0 \sim M_4^2/M < M$. Hence the backreaction will be small, and the larger the horizon generated at the leading order, the smaller the higher-order corrections outside it.

We stress that the quantum dressing of the conical singularity is in fact completely independent of the AdS/CFT correspondence. It happens for *any* 2+1 CFT that couples to 2+1 gravity, independently of whether its ('t Hooft) self-coupling is strong or weak. While ref. [28] claimed that when $g_* = 1$ the solutions (3.1) are never reliable, because of large quantum corrections outside of the horizon, this is true only in the regime of small masses. In the limit $M \to M_{\max}$ the horizon becomes arbitrarily large, (3.6), and the solution (3.1) *is* a black hole. The main feature here is that the regime of intermediate mass states disappears as $g_* \to 1$ because $\mu_{UV} \to M_4 \sim M_3$, and the transition between light states and black holes is sudden. Adding a large number of degrees of freedom expands down to $M_3/\sqrt{g_*}$ the range of masses where the horizons can be trusted and makes quantum cosmic censorhip more efficient. Note that these quantum corrected black holes have a large entropy ($\propto$ the area in the bulk, not on the brane [14]), and that at first sight its origin may be puzzling, considering the fact that the classical background which gave rise to this was modeled as a cone sourced by a point-like distribution of CFT energy. However, this source should really be viewed not as an individual state but as a lump of many CFT degrees of freedom, whose entropy is resolved with the help of gravity and quantum corrections.

Therefore the CFT objects fall into three classes as a function of their mass:

1. Light states with masses $M < \mu_{UV}$ with $\lambda_C \gg r_0$, and so they cannot be reliably described by (3.1). They require a quantum-mechanical description in the bulk independently of the localized 2+1 gravity, and on the AdS_4 side are just the perturbative massive KK modes [1].

2. Intermediate mass objects $\mu_{UV} < M < M_4$, with $\lambda_C > r_0$, and so they too are not black holes. Since their masses are above the cutoff, they cannot be described as bulk KK modes on the AdS_4 side. They are new nonperturbative states, which are bulk deformations of AdS_4. Their detailed properties are sensitive to the physics

at the cutoff scale. If the only new mode which appears at the cutoff is $2+1$ gravity (a non-dynamical mode), they can be viewed as bound CFT states, which may however receive large bulk quantum corrections that are not automatically under control because $\lambda_C/r_0 > 1$.

3. Heavy objects $M_4 < M \leq M_{\max}$ with $\lambda_C < r_0$, and so they really are black holes. As with the intermediate mass states, the description of the black holes with $M_4 < M \ll M_3$ requires physics at distances shorter than the CFT cutoff L, which may be completely reliable if the only new mode at the cutoff is the $2+1$ gravity. Then both the $2+1$ corrections from the graviton and the bulk quantum corrections remain small since they are proportional to $T/M = \hbar/r_0 M < 1$, as seen above. These black holes are unstable to the emission of Hawking radiation, which on the bulk side is a one-loop effect, corresponding to non-planar diagrams in the CFT dual.

The emergence of the new short distance scale $\ell_4 = \hbar/M_4 \ll L$ is analogous to the emergence of very short distance scales $\ell_* = g_S \ell_S$ in string theory, which can be probed by solitonic objects — the D-branes [26].

In closing, we define how to take the classical limit for the $2+1$ theory in a way in which the black holes survive. To identify the appropriate limit, observe from (3.7) that to keep the horizon finite we must take simultaneously $\hbar \to 0$ and $g_* \to \infty$, with $\hbar g_*$ finite. Since also $L = \hbar g_* G_3$ and $G_4 = \hbar/M_4^2 \sim LG_3$ stay finite, the bulk description remains valid. Consider now the black hole entropy $S = \pi g_* x^2/(2+3x)$ and the temperature $T = \mu_{UV}/[4\pi x\sqrt{1+x}]$. Since x is a function of only $G_3 M$ through (3.2), S and T are written in terms of $2+1$ quantities only. Both are formally independent of $\hbar$, and naively seem to remain constant as $\hbar \to 0$. However, taking also $g_* \to \infty$, the black hole temperature vanishes and its entropy diverges, as they should.

4. Quantum black holes in $2+1$ dimensions with $\Lambda_3 < 0$

Due to the peculiarities of $2+1$ gravity, in the previous example the black hole horizon arises only after the leading quantum corrections are included. Hawking radiation and its backreaction will not appear until the next order, which is difficult to compute. By contrast, classical gravity in $2+1$ dimensions with a negative cosmological constant admits not only the conical spacetimes of point particles, but also classical (BTZ) black holes [27]. Spacetimes with a negative cosmological constant can also be constructed as AdS bulk geometries ending on negatively curved branes if their tension does not satisfy the RS2 fine-tuning [30]. Black holes on negatively curved 2-branes in AdS_4 have been constructed in [15], so we can use these solutions to study further our conjecture.

However, the bulk geometry at large distances from negatively curved branes differs in important ways from the bulk surrounding the flat branes discussed previously. The proper size of radial slices decreases away from the brane until a minimal size, a throat, is reached, after which the space re-expands again. Therefore the total bulk volume is infinite. Because of this, the solutions with horizons can be either black holes localized on the brane, or black strings stretching all the way through the AdS space, depending on their mass. A second, positive tension, regulator brane may or may not be introduced to cut this volume off. If the regulator is included, then the relationship between G_3 and G_4 changes to [15]

$$G_3 = \frac{1}{2\sqrt{\lambda}\, L_3} G_4 \,, \tag{4.1}$$

where L_3 is the length scale of the brane cosmological constant, $\Lambda_3 = -1/L_3^2$, and λ is a dimensionless parameter defined by

$$\lambda \equiv \frac{L^2}{L_3^2 - L^2} \,. \tag{4.2}$$

If the brane is only slightly curved, $L_3 \gg L$, i.e., $\lambda \simeq L^2/L_3^2 \ll 1$, we recover (2.2) approximately. The duality as described in section 2 can not be applied in a straightforward manner: the holographic dual is modified in the infrared, and is considerably less understood than in the case of flat branes [31, 32, 33]. Essentially, in this case the presence of the brane that breaks conformal symmetry in the UV communicates the breaking to the IR as well. This can be easily seen on the bulk side. Consider the setup with a regulator brane on the other side of the throat. This ensures the validity of $2+1$ gravity at all length scales, but it alters the CFT in the IR by introducing an IR cutoff. The CFT states fall into a discrete spectrum, with a mass gap that scales as the IR cutoff, $\mu_{IR} \sim \hbar/L_3$. In the limit when the regulator is removed, the gap does not disappear: the fluctuating bulk modes, which correspond to the CFT states, must obey Dirichlet boundary conditions at the AdS boundary to remain normalizable. Thus the presence of the AdS brane leads to a two-sided boundary value problem and the spectrum remains quantized.

The mass gap suppresses Hawking emission for very cold, small black holes, because their temperature is below the gap and so the CFT modes cannot be emitted as thermal radiation. Then, to leading order the backreaction for these would be very suppressed as long as the temperature is below the gap. Other consequences of the mass gap will be apparent near the end of this section. In the following we will work in the approximation where λ is small, so the IR and UV regulators are well separated and (2.2) remains approximately valid.

Besides Hawking emission, we expect quantum corrections from the Casimir effect induced, as in the previous section, by the identifications of points in the background. In the cases where the horizon is absent (or has zero temperature) at the classical level, the thermal Hawking radiation will be absent. But for a BTZ black hole, it is difficult to distinguish between thermal and Casimir effects. Actually, the distinction is rather artificial, since both arise from the same non-trivial identifications of points in AdS_3.

We begin the analysis with the solution for a localized black hole on a negatively curved 2-brane found in [15],

$$ds^2_{\rm brane} = -\left(\frac{r^2}{L_3^2} - 8G_3M - \frac{r_1(M)}{r}\right)dt^2 + \left(\frac{r^2}{L_3^2} - 8G_3M - \frac{r_1(M)}{r}\right)^{-1} dr^2 + r^2 d\varphi^2 \,, \tag{4.3}$$

which is asymptotic to AdS_3. This is similar to the BTZ black hole of mass M, with an extra term $r_1(M)/r$. As in the previous example, $r_1(M)$ can only be given in parametric form. Defining a parameter z via

$$G_3 M = \frac{z^2(1+z)(\lambda - z^3)}{2(\lambda + 3z^2 + 2z^3)^2}\,, \tag{4.4}$$

then

$$r_1 = 8L_3\sqrt{\lambda}\,\frac{z^4(\lambda + z^2)(1+z)^2}{(\lambda + 3z^2 + 2z^3)^3}\,. \tag{4.5}$$

The range of masses in (4.3) which do not lead to naked singularities or to delocalization of the black hole into a black string is $-1/8G_3 \le M \le 1/24G_3$ (obtained by varying $z \in [0,\infty)$). For $M = M_{min} = -1/8G_3$ the correction term vanishes, $r_1 = 0$, and one recovers AdS_3 in global coordinates. The range $-1/8G_3 < M < 0$ corresponds, in classical vacuum gravity, to conical singularities, but here they are dressed with regular horizons. In figure 1 we display the bulk horizon area of all these solutions [15]. This helps us identify two branches of solutions: the branch labeled **1** starts at $M = -1/8G_3$ and ends at $M = 1/24G_3$. Branch **2** begins at $M = 0$ and zero area, and ends at the same point as the previous one.

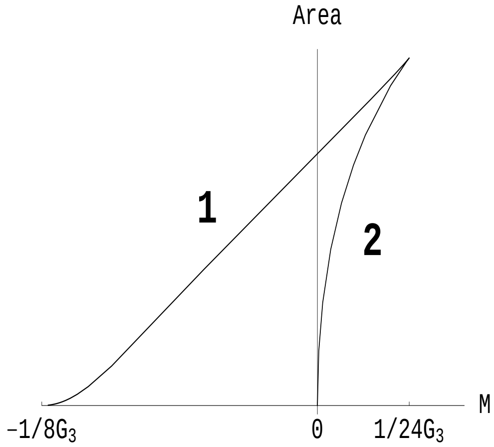

Figure 1: Mass dependence of the 4D area of black holes on an AdS_3 brane.

As before, (4.3) does not solve the vacuum Einstein equations with a negative cosmological constant. Instead, the stress-energy tensor that supports (4.3) contains a correction of the form

$$T^{\mu}{}_{\nu} = \frac{1}{16\pi G_3}\frac{r_1(M)}{r^3}\,\mathrm{diag}(1,1,-2)\,. \tag{4.6}$$

We must discuss how, in accord with our conjecture, these terms encode the quantum effects in the dual theory.

The sector $-1/8G_3 \le M < 0$ of the first branch is naturally interpreted as in the previous section: these solutions are classical conical spacetimes dressed with a horizon from the backreaction of the Casimir energy of the CFT. We are not aware of any calculations of the Casimir energy of a conformal field in conical ($M < 0$) AdS_3 spacetimes, nor of its backreaction. However, we can verify the correspondence between this sector and the one of the previous section, in the limit where the cosmological constant vanishes, $L_3 \to \infty$. If we take this limit for the solutions (4.3) and rescale the time and radial variables to their canonical form at infinity, we find

$$ds^2_{\mathrm{brane}} \to -\left(1 - \frac{r_1}{(8G_3|M|)^{3/2} r}\right)dt^2 + \left(1 - \frac{r_1}{(8G_3|M|)^{3/2} r}\right)^{-1} dr^2 + 8G_3|M| r^2 d\varphi^2\,. \tag{4.7}$$

This has the same form as (3.1), with r_0 identified as $r_1/(8G_3|M|)^{3/2}$. The mass of the limiting solution, $\tilde{M}$, obtained from the conical deficit in (4.7), is

$$\tilde{M} = \frac{1}{4G_3}\left(1 - \sqrt{8G_3|M|}\right) . \tag{4.8}$$

The masses in asymptotically flat and AdS spaces are differently measured, so it is not surprising that $\tilde{M}$ differs from M. What is important is that the range of masses $-1/8G_3 \le M \le 0$ maps precisely to the range in asymptotically flat space, $0 \le \tilde{M} \le 1/4G_3$. One can also check that in the limit $L_3 \to \infty$, $r_1/(8G_3|M|)^{3/2}$ as a function of $\tilde{M}$ becomes exactly the same as r_0 in (3.2) and (3.3), with the identification $z \to 1/x$. Hence we are quite confident that this sector of AdS_3 solutions can be interpreted as Casimir-censored singularities, and where the censorship is reliable for sufficiently large masses $\tilde{M}$, as before.

In the sector $0 \le M \le 1/24G_3$ there are two branches of black holes. For a given mass, branch 1 solutions have larger area than branch 2. We will see that the interpretation is clearer for the solutions in branch 1.

For a conformally coupled scalar at weak coupling residing in the BTZ background, the renormalized stress tensor $\langle T^\mu{}_\nu\rangle$ has been calculated in [4, 5, 6], and it has the same structure as (3.8),[3] now with

$$\alpha(M) = \frac{(8G_3M)^{3/2}}{16\sqrt{2}\pi}\sum_{n=1}^{\infty}\frac{\cosh 2n\pi\sqrt{8G_3M}+3}{\left(\cosh 2n\pi\sqrt{8G_3M}-1\right)^{3/2}} . \tag{4.9}$$

Since this $\langle T^\mu{}_\nu\rangle$ has the same structure as the brane stress-energy tensor (4.6), the back-reaction calculated in [5, 6] results in a geometry like the brane metric (4.3).

This stress-energy tensor is not of the thermal type $\propto \mathrm{diag}(-2,1,1)$. However, this does not conflict with the fact that the CFT in the presence of the black hole is in a thermal state. Ref. [5] showed that the Green's function from which this $\langle T^\mu{}_\nu\rangle$ is derived is periodic in imaginary time, with a period equal to the local Tolman temperature dictated by the black hole. Moreover, this Green's function satisfies the analyticity properties that characterize the Hartle-Hawking state. This means that there is a thermal component in the stress-energy tensor of the CFT, in static equilibrium with the black hole. The fact that the tensor structure of $\langle T^\mu{}_\nu\rangle$ does not conform to the canonical thermal one near infinity reflects the presence of a large Casimir contribution.

For the $M=0$ black hole, which has zero temperature in the classical limit, one would expect that the backreaction from Hawking radiation is absent at one loop. In this limit

$$\alpha(0) = \frac{\zeta(3)}{16\pi^4} , \tag{4.10}$$

which is finite and parametrically $\mathcal{O}(1)$, i.e., not small. This indicates that the quantum-corrected solution undergoes a large Casimir backreaction and cannot be the massless

[3] The form of $\langle T^\mu{}_\nu\rangle$ depends on the boundary conditions at the AdS_3 boundary. The brane solutions appear to automatically select 'transparent' boundary conditions, while ref. [5] considers instead Dirichlet or Neumann conditions. The results for transparent conditions follow by omitting all terms in [5] with a "±" factor, bringing [4, 5, 6] into agreement.

zero-area solution in the second branch, but rather the black hole in the first branch, of finite size. For this state

$$r_1(0) = \frac{8}{27} L = \mathcal{O}(1)\ \hbar g_* G_3\,, \tag{4.11}$$

i.e., $r_1(0)/G_3 = \mathcal{O}(1)\ \hbar g_* \alpha(0)$, and so the brane and CFT stress-energy tensors agree and the interpretation is consistent. The same is true for all $M > 0$ black holes in the first branch: the dependence of r_1 on M is weak when $\lambda \ll 1$, so $r_1(M)$ remains $\sim L$, and similarly $\alpha(M) = \mathcal{O}(1)$ in the range of masses $0 < M < 1/24G_3$, so we find the same agreement up to numerical factors. It is difficult to compare the mass dependence with the same level of rigor as in the asymptotically flat case. For example, the fermions are typically much more sensitive to the cosmological constant than scalars, and so the details of the mass dependence of the function $\alpha(M)$ for the complete dual CFT, even if we ignore the effects of strong coupling, will be quite different from the scalar contribution (4.9). For the largest possible masses, $M \approx 1/24G_3$, the temperature of the black hole is of the order of the IR cutoff, $\sim \hbar/L_3$, and hence Hawking radiation is not suppressed. One may say that it becomes comparable to the Casimir energy, but it is difficult to tell one from the other.

Therefore, all branch 1 solutions at least fit consistently with our conjecture. The black holes of branch 2 may also allow an interpretation as follows. In our conjecture, no specification is made of what is the vacuum state of the CFT. In particular, the calculation of $\alpha(M)$ in [4, 5, 6] was performed assuming that the state in which $\langle T_{\mu\nu}\rangle$ vanishes is the global AdS_3 vacuum. However, it is also possible to regard the $M = 0$ state of zero area as a consistent vacuum, in which case the stress tensor would be renormalized so that $\langle T_{\mu\nu}\rangle_{M=0} = 0$. This $M = 0$ black hole would remain uncorrected, and the BTZ black holes with backreaction from a CFT state above this vacuum would result in a branch of solutions starting at zero area at $M = 0$, just like branch 2. While it is difficult to test this idea further, it is tempting to speculate with the possibility of a decay of the $M = 0$ vacuum by making a transition to the more entropic $M = 0$ state of branch 1, followed by evaporation down to the global AdS_3 vacuum.[4]

Finally, we comment on the solutions with masses $M > 1/24G_3$, which also exist when $\Lambda_3 < 0$. The metric they induce on the brane is precisely BTZ without any corrections. In the bulk, these black holes are in fact black strings that stretch beyond the throat region, all the way to the AdS boundary on the other side. Therefore they are extremely sensitive to the infrared modifications in the dual picture, and their full dynamics is clearly not amenable to the description in terms of only $2+1$ CFT+gravity theory. While the apparent absence of quantum corrections to these black holes seems puzzling, a possible resolution is that these black holes are so massive that the backreaction on them is not only small, but even vanishing at the level of planar diagrams. Note that the one-loop stress-energy tensor of the CFT at weak coupling becomes exponentially small in $\sqrt{M}$ for large M (see (4.9)), which may be an indication of such behavior. Another indication comes from the higher-dimensional nature of these solutions: since they extend through

[4]In the presence of supersymmetry, these two vacua differ in the periodicity conditions for fermions, as NS or R vacua, and therefore fall into different superselection sectors.

the throat, these solutions cannot be described by $2+1$ gravity. Instead, for them the $2+1$ gravity effectively decouples, and their temperature should be viewed as a purely bulk loop effect, with $G_3 M$ reinterpreted as $G_4 m$, where m characterizes the mass per unit length of the string. We postpone a detailed consideration of these solutions for future work.

5. Resolving the mystery of the missing $3+1$ black hole

We now turn to the Randall-Sundrum model [1], defined by a single 3-brane in the AdS_5 bulk. We have far less control over the theory now: on the one hand, gravity in $3+1$ dimensions is dynamical; on the other hand, the absence of exact solutions makes the identification of CFT states difficult. Let us proceed by analogy with the $2+1$ analysis. In that case black holes of horizon size $r_H = r_0 < L$ are approximately spherical four-dimensional black holes in the bulk. This feature extends to higher dimensions. Quite generally, a black hole of size r_H on the brane has an extent into the bulk $r_B \sim L \ln(1 + r_H/L)$, so at distances $r_H < L$ the bulk solution becomes progressively less flattened around the brane and rounder, $r_H \sim r_B$. In the present context, it is well approximated, near the horizon, by a five-dimensional Schwarzschild solution. As r_H becomes smaller than L an increasing number of CFT modes in the UV must be interpreted as bulk gravity in order to encode the bulk geometry. Then it is not meaningful to describe the state as a CFT-corrected 3+1 black hole. The situation in $2+1$ dimensions was in this regard better than one had any right to expect, since the picture of a classical solution, the conical singularity, dressed by CFT corrections was actually valid for masses all the way down to the scale $M_3/\sqrt{g_*} \sim M_4$, i.e., distances *much smaller* than the CFT length cutoff $\hbar/\mu_{UV} \sim L$. The reason is that pure classical gravity in $2+1$ dimensions is topological, so the CFT corrections give the leading dynamical effects of gravity. In that case, the length scale $r_0 \sim L$ does not determine any parametrically new mass scale.

Instead, in $3+1$ dimensions the transition point defined by the equality $r_H \sim L \sim G_4 M \sim (G_5 M)^{3/2}$ determines, through (2.1), (2.2) and (2.3), the new mass scale $\sqrt{g_*} M_4$. We can not sensibly describe black holes lighter than this as CFT-corrected $3+1$ black holes. Nevertheless, the bulk description holds as long as the backreaction in the bulk remains small. This is the case if $M > M_5 \sim M_4/g_*^{1/6}$. This suggests that the small black holes above this scale are additional states of the CFT, besides the light modes of mass $M < \mu_{UV}$. However, since they are very sensitive to the UV regulator of the CFT, they are not suitable for testing our conjecture. Only for $M > \sqrt{g_*} M_4$ can the light bulk KK modes be consistently interpreted as modes of a CFT and not as gravity.

Therefore, in what follows we will focus on black holes with mass $M > \sqrt{g_*} M_4$, i.e., size $r_0 > L$. Since their mass is much greater than M_4, the backreaction of $\langle T_{\mu\nu} \rangle$ can be regarded as a small perturbation of the classical black hole solution and treated order by order as an expansion in $\hbar$. In general, $\langle T_{\mu\nu} \rangle$ depends on the definition of the quantum vacuum in a crucial way [3, 34]. There are three usual choices, each describing a different physical situation:

1. The Hartle-Hawking state, which describes a black hole in a thermal bath in equilibrium with its own radiation. The state of the CFT is regular at the event horizon. Far

from the black hole $\langle T_{\mu\nu}\rangle$ describes a gas of 4D CFT radiation at the Hawking temperature. This is incompatible with asymptotic flatness. A natural possibility is that a small backreaction results in an FRW universe containing a black hole immersed in thermal radiation.

2. The Unruh state, which describes the process of black hole evaporation. The stress-energy tensor is regular only at the future horizon, and there is a thermal flux of radiation at future null infinity. Consistent backreaction must produce a time-dependent, quantum-corrected, evaporating black hole solution.

3. The Boulware state, which describes a static configuration, with a stress-energy tensor that vanishes at infinity but diverges at the horizon. The backreaction effects convert the horizon into a null singularity. This singularity can be cut away by a static interior solution if it is greater than the singular surface, such as a star.

According to our conjecture, the solution for a black hole on the RS2 brane must correspond to one of these choices. It is now obvious why the search for a static, asymptotically flat black hole solution on the brane has been fruitless so far: the state 1 is not asymptotically flat, 2 is not static, and 3 does not have a regular horizon. The physical reason why we expect that the black hole should sense the backreaction is easy to see from AdS/CFT. As long as the bulk has asymptotic AdS_5 geometry, on the dual side the conformal symmetry of the CFT is valid in the infrared, and so there is no mass gap separating the CFT modes from the vacuum. Any black hole at a *finite* temperature will therefore emit CFT modes with a thermal spectrum, which is precisely the Hawking radiation.[5] On the bulk side, this must be described by a deformation of the bulk geometry near the brane, which arises because the black hole appears as a source in the classical bulk gravity equations.

We should recall here some proposals for static black hole solutions on the brane. For reasons that will become clearer later, such solutions typically become singular in the bulk, so they are not physical. A prototype for this sort of singular behavior is the black string of [8]. Although the brane metric is perfectly regular, there is a divergence of the curvature at the Cauchy horizon in the bulk.

The preceding discussion naturally leads us to considering a radiative solution as the leading-order description of the exterior of a black hole localized on the brane. The detailed description of this geometry on the bulk side would require either the exact bulk solution, which has been missing so far, or a much better approximation than the existing ones. On the side of the $3+1$ CFT+gravity, a description at the same level of rigor would require a careful backreaction analysis, where we should start with a classical Schwarzschild black hole and perturb it by means of the $\langle T_{\mu\nu}\rangle$ in the Unruh state evaluated in the

[5]In the case of RS2 in AdS_5 a step towards the ideas presented here was entertained by T. Tanaka [35], and, simultaneously, by R. Maartens and one of us (NK) in the discussions reported in [36], in order to explain the results of [9]. A naive argument that the bulk dynamics encodes the backreaction from Hawking radiation would lead one to expect that *all* asymptotically flat brane-localized black holes are time-dependent. This would be in conflict with the exact static $2+1$ solutions of [14, 15]. Our conjecture that the classical bulk dynamics encodes *all* quantum corrections at the level of planar diagrams completely resolves this conflict. These exact solutions in fact strongly support the conjecture.

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classical background geometry. This analysis rapidly becomes quite involved, because of the necessity for describing the near and far field regions of the black hole differently: a negative energy density flux near the horizon, well approximated by an ingoing Vaidya metric; the asymptotic infinity approximated by an outgoing Vaidya metric, and a complicated geometry describing the transition between these asymptotic forms in between. The far-field outgoing metric encodes the flux of Hawking radiation pouring out of the black hole, which is described by the stress-energy tensor

$$T_{\mu\nu} = \frac{\mathcal{L}(u)}{4\pi r^2} \nabla_\mu u \nabla_\nu u \,, \tag{5.1}$$

where u is the retarded null coordinate and $\mathcal{L}(u)$ is the flux luminosity. The perturbed geometry is

$$ds^2 = -\left(1 - \frac{2G_4 M(u)}{r}\right) du^2 - 2dr du + r^2 d\Omega_2 \,, \tag{5.2}$$

where $\frac{dM(u)}{du} = -\mathcal{L}(u)$. To check our conjecture, we should recover the relation between $\mathcal{L}$ and M from leading-order corrections to the black hole geometry induced from the bulk. To make any such calculation precise, we should relate the far-field solution (5.2) to a near horizon one, and then match this solution to the interior. The matching conditions will give the precise form of the relationship between the luminosity $\mathcal{L}$ and the interior parameters.

In order to circumvent the details of the matching between the near and far regions, we resort to a simpler, heuristic calculation that allows us to reproduce the correct parametric dependence of the luminosity. Consider the radiative collapse of a large dust cloud. Match this collapsing cloud of dust, whose dynamics is determined in [9] by a leading-order bulk calculation, to an outgoing Vaidya metric (5.2), following the work of [37].[6] The quantum correction terms propagate through the matching regions, and this relates the outgoing flux of radiation to the *subleading* correction in the interior star geometry, which is $\propto$ $(G_4 ML)^2/R^6$, as calculated in [9, r.h.s. of eq. (6)] (we only consider the limit $Q = \Lambda = 0$ of this equation, which is sufficient for our purposes). Comparing to (5.1) we find $\mathcal{L} \sim G_4(ML)^2/R_0^4 \sim \hbar g_*(G_4 M)^2/R_0^4$, where R_0 is the radius of the matching surface. For a large collapsing mass, this will be near $2G_4 M$, so $\mathcal{L} \sim \hbar g_*/(G_4 M)^2$. This is the value that corresponds to a flux of Hawking radiation of $\sim g_*$ degrees of freedom of the CFT, at a temperature $T_H \sim \hbar/(G_4 M)$, as required. Replacing $M(u)$ by M is consistent since $\mathcal{L} \propto \hbar$ and we are working in an expansion in $\hbar$. Within this approach we cannot obtain a detailed formula with accurate numerical coefficients, but it does reproduce the correct scalings with the black hole and CFT parameters, in complete accord with our conjecture. A more detailed analysis recovering the precise form of the matching conditions would be useful, since it can display how the outgoing flux is turned on as a function of time.

What remains is to verify the consistency of the matching of geometries across the horizon. A simple way to check this is to compare the quantum trace anomalies of the backreacted states in the exterior and interior. The trace anomaly of the quantum stress

[6] This appears in ref. [38], who, however, had an outgoing Vaidya metric everywhere outside the collapsing sphere, and also continued matching this solution to a Reissner-Nordstrom geometry very far away. This latter step seems dubious, because this geometry is very likely singular in the bulk.

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tensor is a local geometric quantity independent of which vacuum the field is in [39, 40]. It has been studied in detail in the AdS/CFT context [41], and in particular in the case of AdS braneworlds in [42, 43, 44]. It gives us further insight into our problem, in that it provides a simple leading-order consistency check, which a configuration must pass in order to be described by the leading-order effects in the duality pair.

For a weakly coupled CFT in $3+1$ dimensions the trace anomaly $\langle T^\mu{}_\mu\rangle$ is, to leading order, [3]

$$\langle T^\mu{}_\mu\rangle = \frac{\hbar}{(4\pi)^2}(aC^2 + bE + c\nabla^2 R)\,, \tag{5.3}$$

where $C^2 = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 2R_{\alpha\beta}R^{\alpha\beta} + R^2/3$ is the square of the Weyl tensor, $E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\alpha\beta}R^{\alpha\beta} + R^2$ is the Gauss-Bonnet term. The coefficients a, b and c depend on the specific matter content of the theory, and in the case of $D=4$ $\mathcal{N}=4$ SU(N) SYM at large N

$$\langle T^\mu{}_\mu\rangle = \frac{\hbar N^2}{32\pi^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{R^2}{3}\right)\,. \tag{5.4}$$

Note the cancellation of the term $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$. Ref. [41] showed how this anomaly is precisely reproduced from a computation in the AdS_5 bulk. This result is perturbatively identical to the familiar quadratic stress-energy correction terms that appear in the effective long distance $3+1$ gravity equations in AdS braneworlds [45], which can be checked explicitly recalling $g_* \sim N^2$ [42, 43].

If the CFT is deformed by relevant operators the behavior in the infrared changes, and the bulk side of the geometry will be quickly deformed away from the AdS geometry. When this occurs, the anomaly coefficients a, b, c in (5.3) will deviate away from the values they take for $\mathcal{N}=4$ SYM, and generically $a+b \neq 0$, so the anomaly may contain the contributions from $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$. The appearance of such terms implies that the bulk is *not* asymptotically AdS_5; it is very likely that a singularity will appear in the bulk, at some finite distance from the brane.[7] On the other hand, the absence of terms $\propto R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ does not imply that the bulk is asymptotically AdS. An example is a radiation dominated FRW cosmology, with the CFT in a thermal state. In the bulk, this corresponds to an AdS-Schwarzschild solution, where the singularity is hidden by a horizon at a finite distance from the brane [47, 19, 48, 49], although the anomaly vanishes.

We can now reinterpret the 'no-go theorem' of ref. [9] within the CFT+gravity theory. There the authors considered the collapse of pressureless homogeneous dust on a braneworld in AdS_5, and following the standard general relativity routine, they attempted to match this interior to an exterior metric, as opposed to a radiating one as we advocated above. Because the interior geometry was a solution of the AdS_5 braneworld junction conditions, it was guaranteed to satisfy the anomaly equation (5.4). However, the exterior geometry, resembling a deformation of the Schwarzschild geometry, was not required to fulfill these

[7]The exception are the situations where the singularity can be dealt with in a physically motivated manner. For instance, a singularity appears when supersymmetry is broken to produce either a confining phase or a mass gap at some finite scale in the infrared, and its resolution is an interesting problem [46].

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equations, but was tailor-made to satisfy the matching conditions on the envelope of the collapsing dust. Requiring the exterior geometry to be static, ref. [9] found that the Einstein tensor must have a nonvanishing trace in the exterior region equal to

$$G^{\mu}{}_{\mu} = -12L^2 \frac{(G_4 M)^2}{r^6} \,. \tag{5.5}$$

This led [9] to conclude that the exterior geometry can not be static.

The interpretation of this result is that (5.5) is the quantum anomaly induced by the backreaction, which is inconsistent with the anomaly of the interior solution. One can easily check that the trace (5.5) is proportional to $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$. Indeed, the trace anomaly in the Schwarzschild background is [40]

$$\langle T^{\mu}{}_{\mu} \rangle = \hbar \frac{3(a+b)}{\pi^2} \frac{(G_4 M)^2}{r^6} \,, \tag{5.6}$$

which comes entirely from the Riemann-squared term. According to our discussion, the interior and exterior geometries considered in [9] cannot belong in the same theory, even if they were to be both interpreted in the AdS/CFT context. In fact, using the AdS_5/SYM/RS2 relation $L^2 = (4/\pi)\hbar N^2 G_4$ in (5.5) suggests that the exterior theory should have $a + b = -2N^2$. Obviously, such matching is not physically sensible.[8] Instead, one must look for a different exterior, where the metric correctly encodes the quantum backreaction. This naturally leads to a time-dependent evaporating black hole (5.2).

Indeed, the matching to the far-field Vaidya metric (5.2), is consistent with the anomaly check. The tracelessness of the radiation stress-energy implies $R^{\mu}{}_{\mu} = 0$, and so the anomaly vanishes, with no contributions from the $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ terms. Although this argument by itself does not fully guarantee that the bulk will be free from singularities, it passes the anomaly check with only minimal assumptions which are physically well-motivated.

Therefore, barring exotic possibilities, we see that the classical bulk dynamics requires braneworld black holes to be time-dependent. We have arrived at this conclusion by studying only the dynamics projected on the $3+1$ braneworld, but we would also like to understand the picture from the point of view of the full bulk AdS_5 spacetime. Then the following questions arise naturally: (i) What is the bulk dual of the Hawking radiation emitted by the black hole? (ii) Why should a classical black hole on the brane *have to* emit anything? (iii) Why should this emission, which is classical from the point of view of the bulk, appear as *thermal* radiation in the dual 3+1 picture?

The answer to (i) is obvious: in the $3+1$ CFT+gravity theory, Hawking radiation consists of CFT modes, whose dual in the bulk are KK gravitons. The bulk emission consists of classical gravitational waves. To answer (ii) we have to find a natural mechanism that causes the black hole to classically emit these waves into the bulk. Observe that the black hole is moving along with the brane in AdS_5. The brane is a domain wall that is accelerating away from the center of AdS. So the black hole also accelerates, and as a consequence it must emit gravitational waves. This means that the bulk dual of Hawking

[8] Away from the horizon, the matching may be possible as a bubble at the interface between the two phases. This might allow an interpretation of the solutions in [11].

radiation is gravitational bremsstrahlung. It would be interesting to substantiate this qualitative idea with a more detailed analysis of the relevant classical bulk physics. This will also shed light on the important question (iii), which for now is left open. It is encouraging to note that we can at least reproduce the estimate to leading order for the location of the peak of the distribution, determining Hawking temperature, from purely classical considerations in the bulk. Namely, the classical waves which would be emitted into the bulk would have a characteristic frequency determined by the inverse gravitational length of the source, which for a $5D$ theory in the bulk is given by $\omega \sim \sqrt{G_5 \rho}$, where ρ is the energy density of the region where the gravitational waves are emitted, $\rho \sim M/\mathrm{Vol}_4$. Since the black hole is accelerating at a rate $1/L$, we can estimate to leading order $\mathrm{Vol}_4 \sim r_H^3 L \sim G_4^3 M^3 L$, leading to $\rho \sim 1/(G_4^3 L M^2)$, and so using (2.2) we find $\omega \sim 1/(G_4 M)$, i.e. precisely the formula for Hawking temperature! However the complete classical description of the thermal spectrum is yet to be determined. Note that the bulk solution must be time asymmetric, in contrast to the lower-dimensional solutions of [14], where the black hole accelerates eternally and the net flux of radiation vanishes.

Working on the bulk side, one should be able to reproduce the black hole luminosity $\mathcal{L} \sim \hbar g_*/(G_4 M)^2$ by solving classical 5D equations. Indeed, viewing the radiation loss as a classical effect clarifies why this emission rate is so huge. It also explains why the large release of energy into the bulk does not contradict the statement that the black hole radiates mainly on the brane [50]: this applied to Hawking radiation into the bulk, which was compared to Hawking emission of non-CFT modes on the brane, in theories where there may be additional degrees of freedom stuck to the brane. But from the bulk point of view, the large bremsstrahlung emission we are considering is *not* a quantum-mechanical process, and so is not constrained by the analysis of [50]. It must not be confused with Hawking radiation into the bulk, which is a much smaller effect. From the dual CFT+gravity point of view, where the radiation is a quantum phenomenon, the large black hole luminosity is simply a consequence of the large number of CFT modes.

The bulk view would also allow one to follow the evolution of the evaporating black hole beyond the threshold $r_H \sim L$, $M \sim \sqrt{g_*} M_4$ at which the description in terms of a $3+1$ theory of gravity+CFT breaks down, even down to $M_5 \ll M_4$, as we have been arguing above. A black hole of size $r_H \ll L$ is approximated near the horizon by a five-dimensional, *static* Schwarzschild solution. Classical radiation into the bulk, and therefore $3+1$ Hawking radiation into CFT modes, is suppressed for such light black holes. An intuitive understanding of why this happens may be gained from tunneling suppression [51]. While large black holes are shaped like pancakes around the brane, they extend to distances larger than the AdS radius L. Thus they couple to all the CFT modes, including the lightest ones, with M_4 couplings, without any suppressions. On the other hand, while the small black holes are bulging away from the brane, they are much smaller than the AdS radius, and from the perturbative point of view, they live inside the RS2 'volcano'. Hence their classical couplings to all bulk graviton modes are tunneling-suppressed in the sense of [51], and are exponentially weaker than M_4. Thus the radiation rate must go down significantly.[9]

[9]The same effect occurs for a large object on the brane, of mass $M \gg M_4$ but lower density than a black

Hence the light black holes evaporate, although more slowly, via bulk Hawking radiation. It is interesting to ask what would be the bulk description of the final stages of black hole evaporation. While on the CFT+gravity side, the classical area theorems are violated by the quantum effects of Hawking radiation, leading to the shrinking of the black hole horizon, on the bulk side there are no quantum effect to leading order and the bulk version of the area theorems still applies.[10] This would imply that a black hole cannot disappear from the bulk. Hence a consistent picture would be that the disappearance of a black hole in the CFT+gravity theory corresponds to the classical sliding of the black hole from the brane into the bulk. It would be very interesting to verify this explicitly. Since this picture for the evolution of an evaporating black hole is based on specific properties of the UV extension provided by the bulk theory, there is no reason why it should apply to situations that do not have an AdS/CFT dual description.

6. Conclusions

We have proposed here a radical change of perspective on how to view black holes in the context of AdS/CFT correspondence. The previous work on black holes within the AdS/CFT framework has been aimed at understanding a $D+1$-dimensional black hole sitting at the center of AdS_{D+1} in terms of the quantum states of a CFT at the boundary. In this case, the black hole radiates via quantum effects in the bulk, and one expects to learn about the quantum properties of a black hole by studying its dual boundary description.

Instead, we put the black hole itself in the dual theory extended with dynamical gravity. On the bulk side, this is realized by putting the black hole on a brane in the cutoff AdS bulk, which localizes dynamical gravity. Then we can study the quantum properties of a D-dimensional black hole in terms of classical physics in the bulk. The quantum Hawking radiation of CFT modes is described as the emission of gravitational waves into the bulk, and the classical bulk point of view may lead to a better understanding of quantum black hole evaporation. Each of these two approaches prompts different classes of questions, which can be naturally answered within these frameworks.

We have provided strong support for this new point of view with a detailed analysis of the black hole solutions on a 2-brane in AdS_4 and their dual $2+1$ CFT+gravity description. Our analysis has also revealed new features of the states of the $2+1$ CFT coupled to $2+1$ gravity, and has shown explicitly that quantum effects can censor singularities. We have found that the main properties of the quantum censorship mechanism in $2+1$ dimensions are in fact quite general, and should remain valid outside of the context of AdS/CFT. The censorship is however amplified in the presence of many CFT modes, and this appears to be the main requirement that makes the quantum censor efficient.

hole, such as a star. Even if the star had accelerated by being stuck to the brane, the bulk deformation it would cause would have been confined to distances less than L, so its emission would have been tunneling-suppressed. The reason why a black hole radiates in the bulk whereas a star does not is also dual to the problem of the different choices of CFT vacua and boundary conditions for the radiation. This deserves further study.

[10] We thank D. Marolf for very useful discussions on this issue.

In the context of the RS2 model in AdS_5, we have been able to argue why an asymptotically flat, static, regular black hole localized on the brane, could not be found. We emphasize again that while we have been working in the context of AdS braneworlds like RS2, which have proven to be a very useful tool to study black holes, we expect that many of our results should naturally extend to any CFT+gravity theory, even if a dual bulk description along the lines of RS2 does not exist.

There remain a number of open issues. We have given a qualitative argument for why a black hole on a brane should emit classical gravitational waves, but it is still unclear why this emission, which can be analyzed and understood in purely classical terms, should project on the brane as a *thermal* flux of radiation. The problem belongs to a class of connections between classical effects in the bulk and thermal effects in the dual theory. The conventional AdS/CFT approach tried to understand how a state of the CFT encodes the classical causal structure of the bulk black hole. The present problem is quite different and could be an easier one, since we may have some hope of analyzing the classical bulk physics involved in the radiation.

An aspect of our conjecture that we have only barely touched upon is the choice of vacuum of the CFT. This is closely related to understanding Hawking radiation as classical bulk bremsstrahlung. It would be natural to expect that each consistent choice of vacuum should correspond to a specific bulk AdS solution, which differ from each other by the boundary conditions for the bulk waves at the bulk AdS horizons. We have discussed a possible example in the case of BTZ black holes. They admit both the $M = -1/8G_3$ and $M = 0$ states as consistent vacua, which we have conjectured to correspond to the two branches of black holes localized on the brane. In $3+1$ dimensions we also had alternative vacua, but we have only examined the physics related to the Unruh vacuum, which models the late time behavior of the collapse. The bulk dual of a black hole with backreaction from the Hartle-Hawking state would be quite interesting as well: The asymptotic thermal radiation is dual to a large black hole inside the AdS_5 bulk. The motion of a brane in this spacetime generates the radiation-dominated FRW evolution on the brane. Hence the Hartle-Hawking state should be described in the dual bulk theory as a black hole localized on a brane, which is itself moving in the background of a large bulk black hole in the center of AdS_5. The next-to-leading order corrections to the $2+1$ asymptotically flat black holes may lead to a similar picture. On the other hand, the Boulware state should result in a null singularity that is localized on the brane. It would be interesting to check if there exists a relationship between these solutions and the static linearized approximation in the RS2 model [1, 52]. We believe that these questions merit further consideration and hope to return to them in the future.

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